Stress concentrations at an oblique hole in a thick plate

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Summary

An empirical equation has been obtained for the elastic stress concentration factor at an isolated oblique circular- cylindrical hole in thick flat plate subjected to a uniform, arbitrarily oriented, uniaxial tension. The equation is presented and its development is outlined in this note.

Keywords Stress concentration factor; Empirical equation; Surface fit optimisation; Oblique hole; Thick plate.

1. Introduction

An oblique hole in a thick flat plate is shown in Fig.1. The hole is circular-cylindrical and the plate thickness is of the order of the hole diameter or greater; the applied stress is a uniform uniaxial tension in the plane of the plate. The plate edges are assumed to be remote from the hole. The definitive variables are

(i) the angle α between the hole-axis and the plate normal (i.e. the angle of obliquity);

(ii) the ratio of plate thickness (h) to hole diameter (d), h/d;

(iii) the angle ϕ between the direction of the applied stress and the major axis of the elliptic intersection of the hole in the surface plane of the plate.

The results of a parametric photoelastic study of the stress distribution at such a hole were presented in ref.1 for values of the variables h/d =1.33, 2.00 and 3.00 (approx.); ϕ=0, 30º, 60º and 90º; and α=0, 15º, 30º, 45º, and 60º. (For h/d =2.00, the ϕ=30º series was not included.)

Specific stress values were given in non-dimensional form as stress indices i.e. stress divided by the applied stress. The maximum stress index at each hole (i.e. the stress concentration factor), designated I_{1MAX}, is tabulated in ref. 1; values are reproduced here in Table 1 for ease of reference. Values for the case of h/d ≈3.00 are plotted in Fig.2a to illustrate typical trends.

Details of the positions of the maximum stress indices, I_{1MAX}, on the hole surface, are also given in ref. 1. For $\alpha = 0$, the maximum index occurs at the mid–plane position at the ends of the diameter normal to the applied load direction. With increasing obliquity, I_{1MAX} moves progressively from the mid–plane position, until for $\alpha = 60°$ the maximum occurs at the rim of the hole. In general, the position of I_{1MAX} is close to the point on the hole surface where the tangent is parallel to the applied load direction. However, for α values greater than 45°, the position of I_{1MAX} is somewhat closer to the 'acute' end of the major axis of the elliptical intersection (end 'a' in Fig. 1) than the tangent position when ϕ is not equal to 90°.

The development of an empirical relationship covering the Table 1 data is the subject of this note.

1.1 Notation

a, b	coefficients
d	hole diameter
h	plate thickness
I_{1MAX}	maximum experimental stress index
$I_{\alpha\phi}$	maximum predicted stress index
I_{00}	value of $I_{\alpha\phi}$ for $\alpha=\phi=0$
n	sample size

S.D standard deviation of relative differences Δ about mean value $\bar{\Delta}$

α angle between hole axis and plate normal (see Fig.1)

Δ relative difference $(I_{\alpha\phi}-I_{1MAX})/I_{1MAX}$

$\bar{\Delta}$ mean value of Δ within a given data set

ϕ angle between direction of applied stress and major axis of the elliptical intersection of the hole in the surface plane of the plate (see Fig.1).

2. Assumed function

It was assumed that a satisfactory approximation to the experimental data could be obtained in the form of the quantity $I_{\alpha\phi}$ derived from the following function

$$I_{\alpha\phi} = I_{00} + a\cos\phi\,(1-\cos\alpha) + b\sin\phi\,(1-\sec\alpha) \quad (1)$$

The I_{00} term is clearly to be identified with the common α=0 value of the curves in Fig. 2a; the second term (with negative a) is consistent with the monotonically decreasing curve for ϕ=0 and the third term (with negative b) with the monotonically increasing curve for ϕ=90°.

Another function, with the $b\sin\phi(1-\sec\alpha)$ term in equation (1) replaced by $b\sin\phi\tan\alpha$, was also studied, but in general the equation (1) function proved the more satisfactory.

3. Determination of I_{00}, a and b

Values of the three fitting parameters (I_{00}, a and b) in equation (1) were determined for which the sum of the squares of the relative differences between the predicted values ($I_{\alpha\phi}$) and the corresponding actual values (I_{1MAX}) of the stress concentration factor in a particular data set ($\Sigma[(I_{\alpha\phi} - I_{1MAX})/I_{1MAX}]^2$) was a minimum. For this purpose the experimental results and computations were tabulated using Microsoft Excel version 7.0 running on a Pentium PC. The least squares fits were minimised using a Newton–Raphson search. The running time was negligible. The whole exercise entailed approximately 20 runs.

Two approaches were adopted. The first was to determine least-squares fit values of I_{00}, a and b for each of the three separate data sets detailed in Table 1. For the h/d≈3.00 set, as an example, the results were

$$I_{00} = 3.404$$

$$a = -3.047$$

$$b = -2.046$$

Values of $I_{\alpha\phi}$ obtained from equation (1) using these values of I_{00}, a and b are tabulated in Table 2a, and the relative differences $[(I_{\alpha\phi}-I_{1MAX})/I_{1MAX}]$ between corresponding predicted and experimental values are given in Table 2b. It can be seen that these relative differences were within the range +6.1% to –6.0%. The mean value was –0.1% and the standard deviation about the mean (S.D.) was 3.6%.

Least squares fit values of I_{00}, a and b, the relative difference range and the standard deviation of the relative differences are given in Table 3 for each data set.

It has been concluded previously [1] that

> "For a given hole obliquity (α) and a given load direction (ϕ) the values of the principal stress indices (I_{1MAX}….) are not significantly dependent upon the ratio of plate thickness (h) to hole diameter (d) over the range 1.33<h/d<3.0"

With this in mind, as a second approach, least squares fit values of I_{00},a and b were obtained for the average I_{1MAX} values for each α,ϕ combinations in Table 1, and then , using these I_{00}, a and b values, the relative difference distribution, mean and standard deviation for each separate data set were obtained.

Least squares fit values for the average data set were:

$$I_{00} = 3.359$$

$$a = -2.884$$

$$b = -2.238$$

Average experimental values, values obtained from equation (1) using the above values of I_{00}, a and b, and relative differences are tabulated in Table 4. The relative difference range was +6.0% to –6.3%; the mean value was –0.34%, and the standard deviation was 3.7%. The relative difference range, mean and standard deviation for each separate data set, obtained using the same values of I_{00}, a and b, are given in Table 5. It can be seen that the equation

$$I_{\alpha\phi} = 3.36 - 2.88\cos\phi\,(1-\cos\alpha) - 2.24\sin\phi\,(1-\sec\alpha) \qquad (2)$$

provides maximum stress indices which differ from experimental values by no more than +9.0% to –7.8%. The greatest mean relative difference value was –1.6% and the standard deviations were approximately 4%.

To facilitate comparison with experimental data, the variations of $I_{\alpha\phi}$ with α for ϕ=0, 30°, 60°and 90°, as derived from equation (2), are shown in Fig. 2b.

4. Discussion

There are 47 independent data items in Table 1. The distribution of the relative differences between values of $I_{\alpha\phi}$ calculated from equation (2) and corresponding values of I_{1MAX} is shown in Fig 3 (the plotted 'number' is the number of relative differences greater than the indicated percentage value). The average relative difference is –0.5%. 36 of the 47 relative differences (77%) are less than 5% (numerically) and only 6 (13%) are greater than 7%. The combination of α and ϕ at which the extreme difference values tabulated in Table 5 occurred appeared to be randomly distributed; there was no indication of a tendency for the extreme values to occur at a particular co-ordinate combination.

It follows from the form of equation (1) that the gradient of the $I_{\alpha\phi}$ versus α curve is zero at α=0 for all ϕ. It can be shown that for all ϕ values less than $\tan^{-1}$a/b, a second zero gradient (i.e. a turning point or minimum) will also occur in the curve. As a consequence, using the a and b values in equation (2), such minima are predicted in the ϕ =0 and ϕ=30° curves. For ϕ=0 this minimum occurs at α=90°, but since no physical significance can be ascribed to such a hole the deduction is meaningless. For ϕ=30° the minimum is predicted to occur at α=48°. A minimum in the I_{1MAX} curve for ϕ=30° can be seen in Fig.2a in the region of α=45°. (A similar minimum occurred in the h/d=1.33 data [1].) The capability of equation (2) to predict this feature of the experimental data is noteworthy. The minimum $I_{\alpha\phi}$ value for ϕ=30°, α=48° was approximately 6% greater than the value indicated in Fig. 2a.

5. Conclusion

Within the range of variables $1.3<h/d<3.00$ and $\alpha \leq 60°$, the equation

$$I_{\alpha\phi} = 3.36 - 2.88\cos\phi\,(1-\cos\alpha) - 2.24\sin\phi\,(1-\sec\alpha)$$

yields estimates of the maximum stress index (i.e. stress concentration factor) at an oblique hole in a flat plate subjected to a uniform uniaxial tension which differ by no more than 9% from experimental values.

Reference

1. Stanley, P. and Day, B.V., " Photoelastic investigation of stresses at an oblique hole in a thick flat plate under uniform uniaxial tension", J. Strain Anal., 1990,25,157-175

Table 1: Experimental data (I_{1MAX})

ϕ (deg)	h/d	α (deg)				
		0	15	30	45	60
0	1.33	3.31	3.16	2.87	2.52	2.08
	2.00	3.26	3.15	3.12	2.44	1.93
	3.00	3.38	3.20	2.94	2.40[1]	1.99[2]
30	1.33	–	3.23	2.98	2.95	3.30
	2.00	–	–	–	–	–
	3.00	–	3.25	3.26	2.88[1]	3.12[2]
60	1.33	–	3.38	3.60	3.65	4.78[3]
	2.00	–	3.56	3.55	3.71	4.75
	3.00	–	3.47	3.68	3.64[4]	4.20[5]
90	1.33	–	3.73	3.96	4.32	5.64[3]
	2.00	–	3.73	3.88	4.37	5.61
	3.00	–	3.55	3.90	4.52[4]	5.63[5]

Notes

1: h/d = 2.80
2: h/d = 2.59
3: h/d = 1.20
4: h/d = 2.90
5: h/d = 2.57

Table 2a: $I_{\alpha\phi}$ from equation (1) for h/d ≈ 3.00

ϕ (deg)	α (deg)				
	0	15	30	45	60
0	3.40	3.30	3.00	2.51	1.88
30	3.40	3.35	3.21	3.05	3.11
60	3.40	3.41	3.47	3.69	4.41
90	3.40	3.48	3.72	4.25	5.45

Table 2b: $(I_{\alpha\phi}-I_{1MAX})/I_{1MAX}$ x 100 for h/d ≈ 3.00

ϕ (deg)	α (deg)				
	0	15	30	45	60
0	0.7%	3.1%	1.9%	4.6%	-5.5%
30	0.7%	3.1%	-1.6%	6.1%	-0.4%
60	0.7%	-1.6%	-5.6%	1.4%	5.1%
90	0.7%	-2.1%	-4.6%	-6.0%	-3.2%

Table 3: Least squares fit values of I_{00}, a and b, relative difference ranges, mean values and standard deviations

h/d	I_{00}	a	b	Difference range	Mean	S.D.
1.33	3.320	–2.667	–2.323	+7.1% –8.8%	–0.14%	3.88%
2.00	3.361	–2.888	–2.374	+3.6% –7.6%	–0.12%	3.61%
3.00	3.404	–3.047	–2.046	+6.1% – 6.0%	–0.12%	3.62%

Table 4: a) averaged experimental data
b) calculated $I_{\alpha\phi}$ data
c) relative differences (%)

(a)

ϕ (deg)	α (deg)				
	0	15	30	45	60
0	3.317	3.170	2.977	2.453	2.000
30	3.317	3.240	3.120	2.915	3.210
60	3.317	3.470	3.610	3.667	4.577
90	3.317	3.670	3.913	4.403	5.627

(b)

ϕ (deg)	α (deg)				
	0	15	30	45	60
0	3.36	3.26	2.97	2.51	1.92
30	3.36	3.31	3.20	3.09	3.23
60	3.36	3.38	3.47	3.74	4.58
90	3.36	3.44	3.71	4.29	5.60

(c)

ϕ (deg)	α (deg)				
	0	15	30	45	60
0	1.3%	2.9%	-0.1%	2.5%	-4.1%
30	1.3%	2.3%	2.5%	6.0%	0.6%
60	1.3%	-2.6%	-4.0%	2.0%	0.0%
90	1.3%	-6.3%	-5.3%	-2.7%	-0.5%

Table 5: Least squares fit values of I_{00}, a and b, from averaged data, with relative difference ranges, means, and standard deviations

h/d	I_{00}	a	b	Difference range	Mean	S.D.
average	3.360	–2.884	–2.238	+6.0% –6.3%	–0.34%	3.37%
1.33	“	“	“	+7.3% –7.8%	–0.51%	4.41%
2.00	“	“	“	+3.5% –7.8%	–1.59%	3.56%
3.00	“	“	“	+9.0% – 5.8%	+0.21%	4.36%

Figure captions

Figure 1: Oblique hole in plate subjected to uniaxial stress

Figure 2a: Experimental data for $h/d \approx 3.00$

Figure 2b: Maximum stress indices derived from equation (2).

Figure 3: Number of Δ values numerically greater than indicated percentage value
(Note: The values of 20 and 27 at 0% are the numbers of positive and negative relative differences, respectively.)

Figure 1: Oblique hole in plate subjected to uniaxial stress

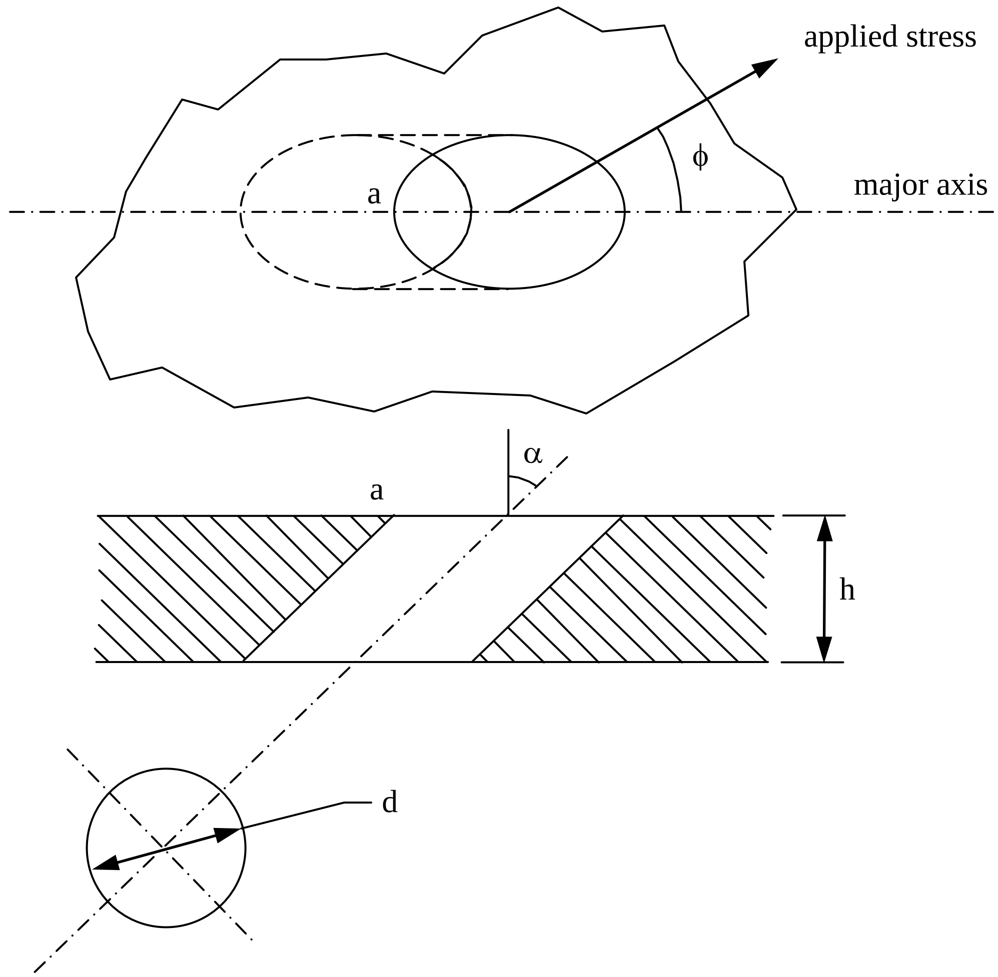


Figure 2a: Experimental data for $h/d \approx 3.00$

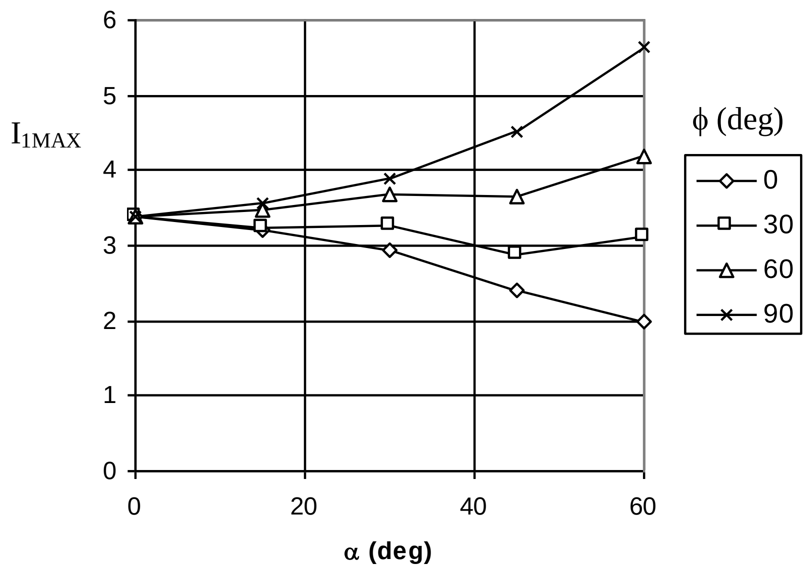
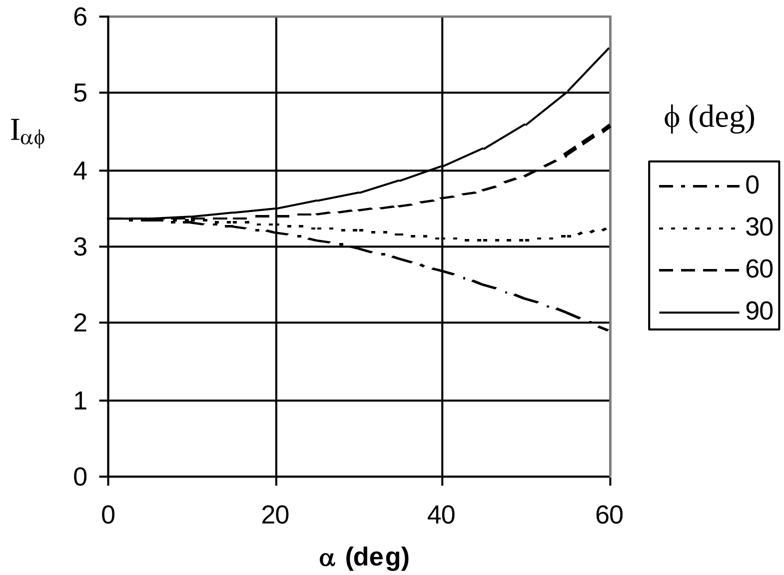


Figure 2b: Maximum stress indices derived from equation (2).



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